Review For Exam 1

The directions for the exam are as follows:

"WRITE YOUR NAME CLEARLY. Do as many problems as you can for a maximal score of 100. Note that you must do at least 10 problems correctly to get 100. Write neatly and legibly in the space provided. SHOW YOUR WORK!"

1. In other words, the exam consists of 10 core problems and 3 extra-credit problems. If you wish, you can do all the 12 problems, but your score will only add up to 100 points. Partial credit will be given.

2. Also remember that you are allowed to use a scientific calculator.

Section 1.1

1. What is a vector? What is the difference between a point and a vector?
2. What is meant by a parametric equation?
3. Find a parametric equation of a line through the point (1, 0, -3) in the direction $-3\mathbf{i} - 10\mathbf{j} + \mathbf{k}$.
4. Find a parametric equation of a line that is <u>parallel</u> to the line in exercise 3, going through the point (1, 1, 1).
5. Can you find a parametric equation of a line that is <u>perpendicular</u> to the line in exercise 3 and going through the point (1, 0, -3)? Is the line that you found the only solution?
6. Find a parametric equation of a line that goes through the points (1, 0, -3) and (6, 7, 6)
7. Are the lines parameterized by $\mathbf{l}(t) = (-2 - t, 3 + t, -2 + t)$ and $\mathbf{p}(t) = (-1 + 3t, 2 + t, -3 + 2t)$ <u>perpendicular</u> to each other? Do they intersect? If they intersect, determine whether the intersection point is also a collision point.
8. Find a parametric equation of the plane through (0, 1, -6) that is spanned by the vectors $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ and $\mathbf{w} = -3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$
9. Let $S = \{(x, y, z); (x, y, z) = (2s - 6t + 1, 3t - s, 7s - 21t + 4), s, t \in (-\infty, \infty)\}$. Is S a plane or a line? Explain.
10. Identify the geometric figure $S = \{(x, y, z); (x, y, z) = (r + 2s - 1, 2s + 3t - 4, r + 3t + 7), r, s \in [0,1], t \in [0,2]\}$.

Section 1.2

1. What is the motivation behind the definition of the dot product?
2. Let $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ and $\mathbf{w} = -3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$. Find the dot product $\langle \mathbf{v}, \mathbf{w} \rangle$.
3. Find the angle between **v** and **w**.
4. Find the unit vector in the direction of **v** and a unit vector in the opposite direction.
5. Find the projection of **v** onto **w**, $P_w(v)$.
6. What do we mean when we say that 2 vectors are linearly dependent? How about 3 vectors that are linearly dependent?

7. State and prove Cauchy-Schwarz and the Triangle inequalities. What does the triangle inequality have to say about space?

Section 1.3

1. What is the motivation behind 2 by 2 and 3 by 3 determinants?
2. What is the volume of a parallelepiped spanned by the vectors **a** = 2**i** + 2**j**, **b** = -3**i** –3**k**, and **c** = 5**j** + 5**k**?
3. Let n be any number. Compute $\begin{vmatrix} n & n+1 & n+2 \\ n+3 & n+4 & n+5 \\ n+6 & n+7 & n+8 \end{vmatrix}$.
4. Suppose $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -4$ and $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 1$. What is $\begin{vmatrix} 3a_{21} & 3a_{22} & 3a_{23} \\ a_{11} & a_{12} & a_{13} \\ b_{31} - a_{31} & b_{32} - a_{32} & b_{33} - a_{33} \end{vmatrix}$?
5. What is the area of a triangle with vertices (1, 1), (3, 3) and (2, 6)?
6. What is the motivation behind the cross product?
7. Compute the cross product of **a** = 2**i** + 2**j** and **b** = -3**i** –3**k.** How is this cross product related to that of –2**b**×**a**?
8. Find a non-parametric equation of the plane (i.e. in terms of x, y, and z) that contains the points (1,1,1), (-1, 0, 1), and (0, 2, 3).
9. Sketch the plane from the above exercise.
10. What is the distance from the point (6, -10, 3) to the plane defined by the equation 3x – 5y + 2z +2 = 0?
11. Given a pair of vectors **v** and **w**, how is the norm of **v**×**w** related to the norms of **v** and **w**?
12. Find the area of the parallelogram spanned by the vectors **a** = 2**i** – **k** and **j** + **k**.
13. Find the distance from the point (1, 2, 3) to the line parameterized by **l**(t) = (t, 2t + 1, t).

Section 1.4

- There will not be any questions on the exam from this section. However, familiarity with Spherical and Cylindrical coordinates will be essential in later sections when we discuss change of variables in integration.

Section 1.5

1. What does the vector e_3 stand for in the space R^5 ?
2. Compute the dot product $a \cdot b$ where **a** = (1, 0, 4, -2) and **b** = (3, 3, -1, 6).

3. Let A = $\begin{pmatrix} 1 & -1 & 1 & -1 \\ 2 & -2 & 0 & 1 \\ 3 & 0 & 0 & 4 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 5 \\ 0 & 4 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$. Compute AB, BA, 3B, A – B, and A^T or state that the operation is undefined.
4. Let C = $\begin{pmatrix} 0 & -1 & 0 & -1 \\ 4 & 8 & 0 & 1 \\ 1 & 0 & 0 & 9 \end{pmatrix}$. Compute A – 2C and A + C.

Section 2.1

1. Sketch or describe the graph of $(x-1)^2 + (y+3)^2 = 4$ in R^2 and in R^3
2. Sketch or describe the graph of $f(x, y) = y^3$
3. Sketch or describe the graph of $f(x, y) = x^2 + 2x + y^2 - 4x - 7$
4. Sketch or describe the graph of $f(x, y) = \dfrac{x^2}{9} + \dfrac{y^2}{25}$
5. Sketch or describe the surface defined by the equation $\dfrac{x^2}{9} + \dfrac{y^2}{25} - z^2 = 0$
6. Sketch or describe the surface defined by the equation $\dfrac{x^2}{9} + \dfrac{y^2}{25} - z^2 = 1$
7. Sketch or describe the graph of $f(x, y) = \ln\left(\dfrac{x^2}{9} + \dfrac{y^2}{25}\right)$
8. Find the equation of the sphere with center (3, -2, 1) and radius 5
9. Sketch or describe the surface defined by the equation $z^2 = 1 - x^2 - \dfrac{y^2}{4}$
10. Sketch or describe the surface defined by the equation $z^4 = (x^2 + y^2)^2$
11. Sketch or describe the graph of $f(x, y) = e^x - y^2$
12. Sketch or describe the graph of $f(x, y) = \begin{vmatrix} x & y \\ y & x \end{vmatrix}$
13. What is the area of the ellipse $\dfrac{x^2}{2} + \dfrac{y^2}{8} = 2$?
14. What is the volume of the ellipsoid $\dfrac{x^2}{4} + \dfrac{y^2}{9} + \dfrac{z^2}{25} = 1$?
15. Suppose S is a 3-D solid with boundary (outline) given by L = {(x, y, z); f (x, y, z) = d}. Let T(x, y, z) = (ax, by, cz) for some positive constants a, b, c. If Vol(S) is the volume of the solid S, What is Vol(T(S)) in terms of Vol(S)? Justify your answer.
16. Sketch or describe the graph of the function $f(x, y) = e^{y+3x}$

Linear Transformations

1. What defines a linear map? What is it intuitively?
2. Let T(x, y, z) = (x + y +z, 2x, 3y – z) and S(x, y) = (-y, x – y, 2y). Compute TS and ST if possible. If not, explain why the operation is undefined. Note that TS is the function composition of T and S. In other words, TS = T○S.
3. Let T(x, y, z) = (x + y +z, 2x, 3y – z) and U(x, y, z) = (x, x + y, x + y –z). Compute 6T, T –2U, and UT.
4. Find the matrices for the linear maps T, S, 6T, T-2U, and UT.
5. Let T(x, y) = (-y, x). Find the inverse of T.
6. What is the inverse of the matrix $\begin{pmatrix} 7 & 3 \\ 2 & 1 \end{pmatrix}$? How about $\begin{pmatrix} -1 & 3 \\ 2 & 3 \end{pmatrix}$? What about $\begin{pmatrix} -2 & -6 \\ 1 & 3 \end{pmatrix}$?
7. Find the linear map that is the inverse of T(x, y) = (7x + 3y, 2x + y). Do the same for S(x, y) = (-x + 3y, 2x + 3y).
8. Secret messages may be encoded in matrix form by replacing a letter by its numeric position in the alphabet. For example "DEAD" becomes the matrix D = $\begin{pmatrix} 4 & 5 \\ 1 & 4 \end{pmatrix}$. To encode a string of words, we will indicate the space between two words by the number 0. In order to prevent unwanted people from reading our message, each encoded message is arranged in a 5 column matrix M and multiplied on the left by the matrix C = $\begin{pmatrix} 7 & 3 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -9 & -4 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix}$ to produce the 5×5 matrix CM.

If CM = $\begin{pmatrix} 43 & 98 & 55 & 0 & 136 & 179 & 63 & 126 \\ 14 & 28 & 17 & 0 & 39 & 53 & 18 & 38 \\ 15 & 20 & 0 & 19 & 13 & 1 & 18 & 20 \\ -53 & -182 & -24 & -240 & -144 & -29 & -126 & 0 \\ 12 & 41 & 6 & 55 & 34 & 7 & 28 & 0 \end{pmatrix}$, decipher the coded message.

Section 2.2

- Evaluate the limit or state that the limit does not exist. In either case, justify your answer.

1. $\lim\limits_{(x,y)\to(1,2)} \dfrac{xy-2x-y+2}{(x-1)^2+(y-2)^2}$

2. $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2y-2xy^2}{x^2+y^2}$

3. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{xyz}{x^2+y^2+z^2}$

4. $\lim\limits_{(x,y,z)\to(2,5,-3)} \dfrac{Sin(4x^2-2y+2z)}{2x^2-y+z}$

5. $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^{2xy}-1}{y}$

6. $\lim\limits_{(x,y)\to(0,0)} \dfrac{Sin(2x)-2x+y}{x^3+y}$

7. $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{2x^2yCos(z)}{x^2+y^2}$

8. $\lim\limits_{(x,y)\to(0,0)} \dfrac{Cos(x)-1-(x^2/2)}{x^4+y^4}$

9. $\lim\limits_{(x,y)\to(0,0)} \dfrac{(x-y)^2}{x^2+y^2}$

10. Use delta epsilon to prove that $\lim\limits_{(x,y,z)\to(2,5,1)} 4x-3y+z=-6$

11. Use delta epsilon to prove that $\lim\limits_{(x,y,z)\to(0,-1,2)} 7x+2y+8z=14$

12. Use delta epsilon to prove that $\lim\limits_{(x,y,z)\to(0,-1,2)}$ (7x + 2y + 8z, x – y – z) = (14, -1)